

Schwartz
11.1 (a)

$$\gamma^5 = -\gamma^0 \gamma^1 \gamma^2 \gamma^3$$

$$(\gamma^5)^2 = -\gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^0 \gamma^1 \gamma^2 \gamma^3$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \Rightarrow \gamma^3 \gamma^0 + \gamma^0 \gamma^3 = 0, \quad \gamma^3 \gamma^0 = -\gamma^0 \gamma^3$$

$$= + \gamma^0 \gamma^1 \gamma^2 \gamma^0 \gamma^3 \gamma^1 \gamma^2 \gamma^3$$

$$\gamma^3 \gamma^1 = -\gamma^1 \gamma^3$$

$$= -\gamma^0 \gamma^1 \gamma^2 \gamma^0 \gamma^1 \gamma^3 \gamma^2 \gamma^3$$

$$= \gamma^0 \gamma^1 \gamma^2 \gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^3$$

$$= \gamma^0 \gamma^1 \gamma^2 \gamma^0 \gamma^1 \gamma^2$$

$$= -\gamma^0 \gamma^1 \gamma^2 \gamma^1 \gamma^0 \gamma^2 = \gamma^0 \gamma^1 \gamma^2 \gamma^0 \gamma^2$$

$$= \gamma^0 \gamma^2 \gamma^0 \gamma^2$$

$$= -\gamma^0 \gamma^0 \gamma^2 \gamma^2$$

$$= (-1)(-1)(1)(1) = \boxed{1}$$

$$(b) \quad \gamma_\mu \not{p} \gamma^\mu = \gamma_\mu \gamma_\alpha p^\alpha \gamma^\mu$$

$$= \gamma_\mu \gamma^\alpha \gamma^\mu p_\alpha$$

$$= \gamma_\mu [2g^{\alpha\mu} - \gamma^\mu \gamma^\alpha] p_\alpha$$

$$= 2\gamma_\mu g^{\alpha\mu} p_\alpha - \gamma_\mu \gamma^\mu \gamma^\alpha p_\alpha$$

$$= 2p - 4p = -2p$$

$$d) \quad \gamma_\mu \not{p} \not{p} \gamma^\mu = ?$$

$$\gamma_\mu \gamma_\alpha \gamma_\beta \gamma_\sigma \gamma^\mu p^\alpha p^\beta p^\sigma$$

$$= [2g_{\mu\alpha} - \gamma_\alpha \gamma_\mu] \gamma_\beta \gamma_\sigma \gamma^\mu \quad \times p^\alpha p^\beta p^\sigma$$

$$= 2 \gamma_\beta \gamma_\sigma \gamma_\alpha - \gamma_\alpha \gamma_\mu \gamma_\beta \gamma_\sigma \gamma^\mu$$

$$= 2 \gamma_\beta \gamma_\sigma \gamma_\alpha - \gamma_\alpha [2g_{\mu\beta} - \gamma_\beta \gamma_\mu] \gamma_\sigma \gamma^\mu$$

$$= 2 \gamma_\beta \gamma_\sigma \gamma_\alpha - 2 \gamma_\alpha \gamma_\sigma \gamma_\beta + \gamma_\alpha \gamma_\beta \gamma_\mu \gamma_\sigma \gamma^\mu$$

$$= 2 \gamma_\beta \gamma_\sigma \gamma_\alpha - 2 \gamma_\alpha \gamma_\sigma \gamma_\beta + \gamma_\alpha \gamma_\beta [2g_{\mu\sigma} - \gamma_\sigma \gamma_\mu] \gamma^\mu$$

$$= 2 \gamma_\beta \gamma_\sigma \gamma_\alpha - 2 \gamma_\alpha \gamma_\sigma \gamma_\beta + 2 \gamma_\alpha \gamma_\beta \gamma_\sigma - \gamma_\alpha \gamma_\beta \gamma_\sigma \gamma_\mu \gamma^\mu$$

$$= 2 \gamma_\beta \gamma_\sigma \gamma_\alpha - 2 \gamma_\alpha \gamma_\sigma \gamma_\beta + 2 \gamma_\alpha \gamma_\beta \gamma_\sigma - 4 \gamma_\alpha \gamma_\beta \gamma_\sigma \quad \times p^\alpha p^\beta p^\sigma$$

$$= 2 \not{p} \not{p} \not{p} - 2 \not{p} \not{p} \not{p} + 2 \not{p} \not{p} \not{p} - 4 \not{p} \not{p} \not{p}$$

$$= 2 \not{p} \not{p} \not{p} - 2 \not{p} \not{p} \not{p} - 2 \not{p} \not{p} \not{p}$$

Now consider $\not{p} \not{p} = \gamma_\alpha \gamma_\beta p^\alpha p^\beta$

$$\gamma_\alpha \gamma_\beta + \gamma_\beta \gamma_\alpha = 2g_{\alpha\beta} \Rightarrow \gamma_\alpha \gamma_\beta = 2g_{\alpha\beta} - \gamma_\beta \gamma_\alpha$$

$$\Rightarrow \not{p} \not{p} = 2p \cdot p - \not{p} \not{p}$$

$$\text{Thus } 2pq - 2rp - 2qp$$

$$= 2[2q \cdot p - rp] - 2p[2q \cdot p - qp] - 2qp$$

$$= \cancel{4pq} - \cancel{2rp} - \cancel{4qp} + \cancel{2qp} - 2qp$$

$$= \boxed{-2qp}$$

$$(d) \quad \{\gamma^5, \gamma^\mu\} = i \left[\gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^\mu + \gamma^\mu \gamma^0 \gamma^1 \gamma^2 \gamma^3 \right]$$

$\gamma^\mu \in \{\gamma^0, \gamma^1, \gamma^2, \gamma^3\}$, so in the process of moving this γ^μ to the right, we will encounter a term where it just commutes, for all other terms, it will anticommute, giving 3 factors of (-1) , $(-1)^3 = -1$, thus we get left and right side in the bracket cancel to give 0.